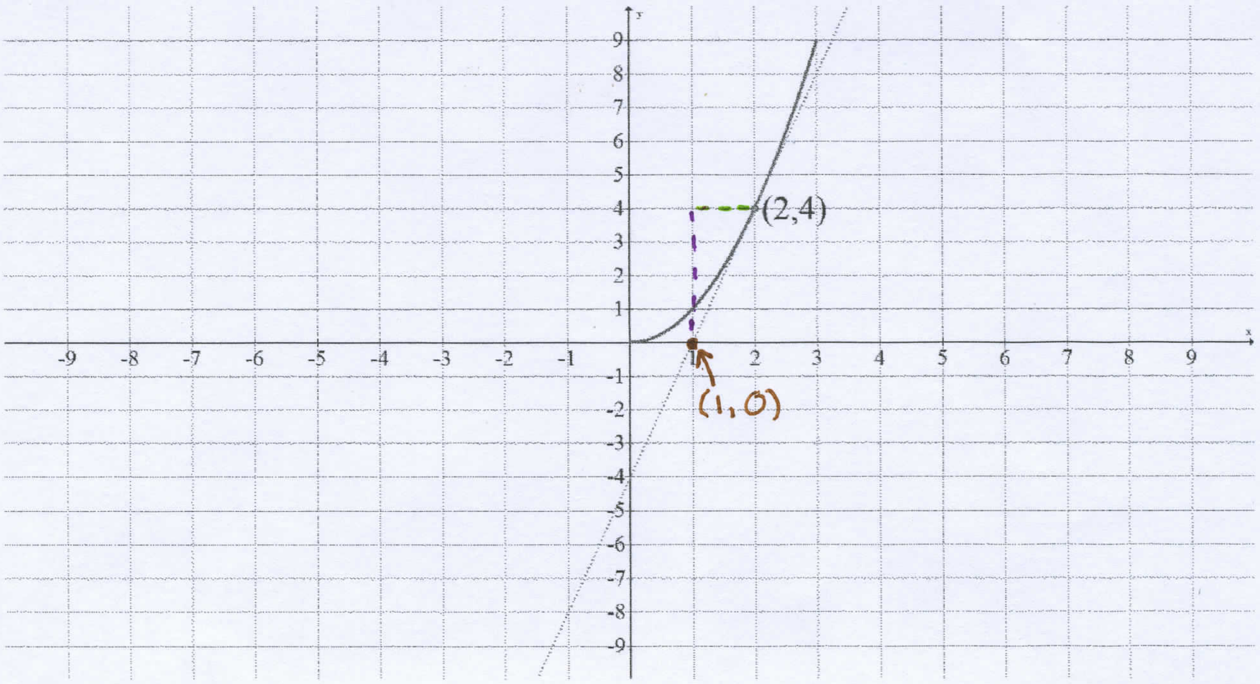
Section 1.5

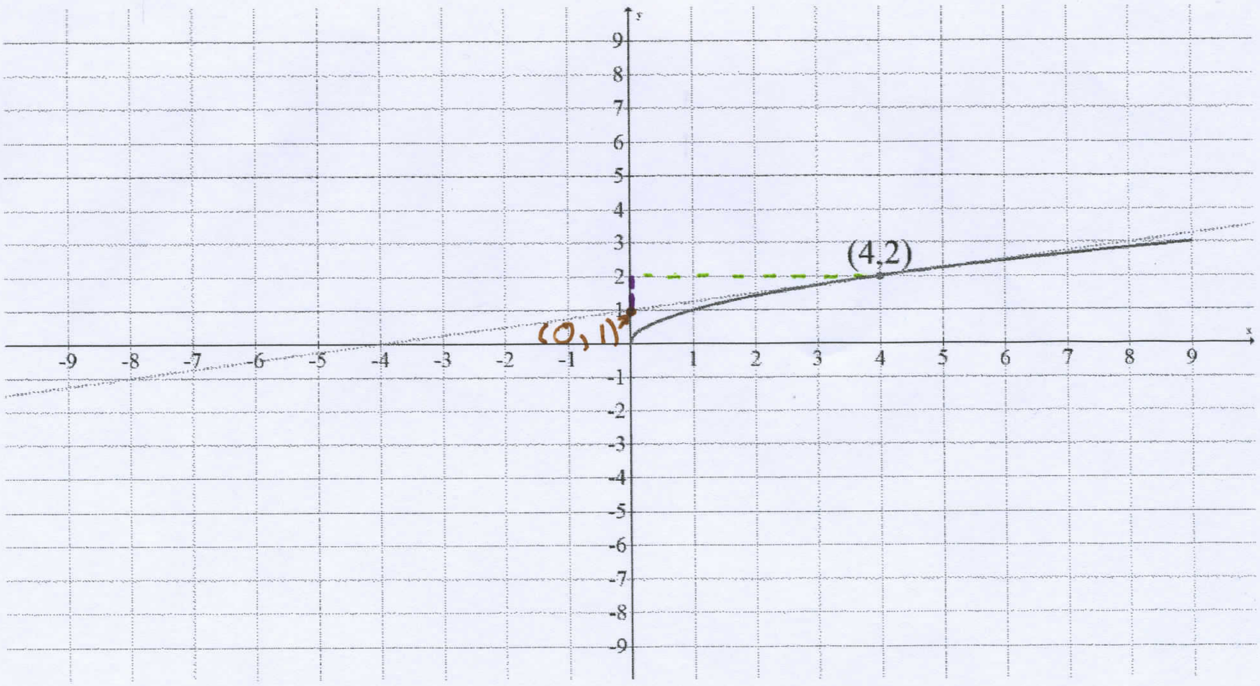
1)



Note: The tangent line is linear and has the same slope between any two points. The slope is the change in the y-value with respect to the change in the x-value. $m = \frac{y_1 - y_2}{x_1 - x_2}$: slope

$$\frac{4-0}{2-1} = \frac{4}{1} = \boxed{4}$$

3)



$$\text{Slope} = m = \frac{y_1 - y_2}{x_1 - x_2} \qquad \frac{2-1}{4-0} = \boxed{\frac{1}{4}}$$

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5. a) $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$f(x) = x^2 + 3x - 4 \qquad f(x+h) = (x+h)^2 + 3(x+h) - 4$

$f(x+h) = x^2 + 2xh + h^2 + 3x + 3h - 4$

$$\lim_{h \to 0} \frac{(x^2 + 2xh + h^2 + 3x + 3h - 4) - (x^2 + 3x - 4)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 + \cancel{3x} + 3h \cancel{- 4} - \cancel{x^2} - \cancel{3x} \cancel{+ 4}}{h} = \lim_{h \to 0} \frac{2xh + h^2 + 3h}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(2x + h + 3)}{\cancel{h}} = \lim_{h \to 0} 2x + h + 3$$

when $h = 0$ for $2x + h + 3$:

$2x + (0) + 3 = 2x + 3$

$$\boxed{f'(x) = 2x + 3}$$

5\. b) $f'(x) = 2x + 3$

$f'(4) = 2(4) + 3 = 8 + 3 = 11$

$\boxed{f'(4) = 11}$

7\. a) $f(x) = 6x^2 + 12$

$f(x+h) = 6(x+h)^2 + 12$

$f(x+h) = 6(x^2 + 2xh + h^2) + 12$

$f(x+h) = 6x^2 + 12xh + 6h^2 + 12$

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$\lim_{h \to 0} \frac{(6x^2 + 12xh + 6h^2 + 12) - (6x^2 + 12)}{h}$

$= \lim_{h \to 0} \frac{\cancel{6x^2} + 12xh + 6h^2 \cancel{+12} \cancel{-6x^2} \cancel{-12}}{h} = \lim_{h \to 0} \frac{\cancel{h}(12x + 6h)}{\cancel{h}}$

$= \lim_{h \to 0} 12x + 6h$ when $h = 0$ for $12x + 6h$: $12x + 6(0) = 12x$

$\boxed{f'(x) = 12x}$

7\. b) $f'(x) = 12x$

$f'(4) = 12(4) = 48$

$\boxed{f'(4) = 48}$

9. a) $f(x) = 3x^2 - 4x + 2$

$f(x+h) = 3(x+h)^2 - 4(x+h) + 2 = 3(x^2 + 2xh + h^2) - 4x - 4h + 2$

$= 3x^2 + 6xh + 3h^2 - 4x - 4h + 2$ $\quad f(x+h) = 3x^2 + 6xh + 3h^2 - 4x - 4h + 2$

$$\lim_{h\to 0} \frac{(3x^2 + 6xh + 3h^2 - 4x - 4h + 2) - (3x^2 - 4x + 2)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{3x^2} + 6xh + 3h^2 \cancel{-4x} - 4h \cancel{+2} \cancel{-3x^2} \cancel{+4x} \cancel{-2}}{h}$$

$$= \lim_{h\to 0} \frac{6xh + 3h^2 - 4h}{h} = \lim_{h\to 0} \frac{\cancel{h}(6x + 3h - 4)}{\cancel{h}} = \lim_{h\to 0} 6x + 3h - 4$$

when $h=0$ for $6x + 3h - 4$:

$6x + 3(0) - 4 = 6x - 4$

$$\boxed{f'(x) = 6x - 4}$$

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9. b) $f'(x) = 6x - 4$

$f'(4) = 6(4) - 4 = 24 - 4 = 20$

$$\boxed{f'(4) = 20}$$

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11. a) $f(x) = \frac{2}{x}$ $f(x+h) = \frac{2}{x+h}$

$$\lim_{h\to 0} \frac{\left(\frac{2}{x+h}\right)\frac{\cdot x}{\cdot x} - \left(\frac{2}{x}\right)\frac{\cdot(x+h)}{\cdot(x+h)}}{h} = \lim_{h\to 0} \frac{\frac{2x-2(x+h)}{x(x+h)}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{\cancel{2x}-\cancel{2x}-2h}{x(x+h)}}{h} = \lim_{h\to 0} \frac{-2\cancel{h}}{x(x+h)} \cdot \frac{1}{\cancel{h}} = \lim_{h\to 0} \frac{-2}{x(x+h)}$$

when $h=0$ for $\frac{-2}{x(x+h)}$:

$$\frac{-2}{x(x+(0))} = \frac{-2}{x(x)} = \frac{-2}{x^2}$$

$$\boxed{f'(x) = \frac{-2}{x^2}}$$

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11. b) $f'(x) = \frac{-2}{x^2}$

$$f'(4) = \frac{-2}{(4)^2} = \frac{-2}{16} = \frac{-1}{8}$$

$$\boxed{f'(4) = \frac{-1}{8}}$$

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13. a) $f(x) = \frac{5}{x}$ $\qquad$ $f(x+h) = \frac{5}{x+h}$

$$\lim_{h\to 0} \frac{\left(\frac{5}{x+h}\right)\frac{\cdot x}{\cdot x} - \left(\frac{5}{x}\right)\frac{\cdot(x+h)}{\cdot(x+h)}}{h} = \lim_{h\to 0} \frac{\frac{5x-5(x+h)}{x(x+h)}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{5x} - \cancel{5x} - 5h}{x(x+h)} \div h = \lim_{h\to 0} \frac{-5\cancel{h}}{x(x+h)} \cdot \frac{1}{\cancel{h}} = \lim_{h\to 0} \frac{-5}{x(x+h)}$$

when $h=0$ for $\frac{-5}{x(x+h)}$:

$$\frac{-5}{x(x+0)} = \frac{-5}{x(x)} = \frac{-5}{x^2}$$

$$\boxed{f'(x) = \frac{-5}{x^2}}$$

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13. b) $f'(x) = \frac{-5}{x^2}$

$$f'(4) = \frac{-5}{(4)^2} = \frac{-5}{16}$$

$$\boxed{f'(4) = \frac{-5}{16}}$$

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15. a) Note: The derivative of a function is an equation for the slope of the function. At the point of tangency, the slope of the function and the tangent line are the same. So we find the derivative of the function.

$f(x) = x^2 + x - 4$

$f(x+h) = (x+h)^2 + (x+h) - 4$

$f(x+h) = x^2 + 2xh + h^2 + x + h - 4$

$$\lim_{h \to 0} \frac{(x^2 + 2xh + h^2 + x + h - 4) - (x^2 + x - 4)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 + \cancel{x} + h \cancel{- 4} \cancel{- x^2} \cancel{- x} \cancel{+ 4}}{h} = \lim_{h \to 0} \frac{2xh + h^2 + h}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(2x + h + 1)}{\cancel{h}} = \lim_{h \to 0} 2x + h + 1$$

when $h = 0$ for $2x + h + 1$:

$2x + 0 + 1 = 2x + 1$

$\boxed{f'(x) = 2x + 1}$

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15. b) Note: To find the equation of the tangent line, we need the slope of the line and one point on that line.

$x=3$

$f(3)=(3)^2+(3)-4$ ← • At a point of tangency the function and the line have the same point.

$f(3)=9+3-4$

$f(3)=8$

$(3,8)$

$f'(x)=2x+1$

$f'(3)=2(3)+1=6+1=7$

$m=7$

$y-y_1=m(x-x_1)$

$y-8=7(x-3)$

$y-8=7x-21$

$\quad +8 \qquad\quad +8$

$\boxed{y=7x-13}$

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17. a) $\underline{f(x) = 3x^2 + 7}$

$f(x+h) = 3(x+h)^2 + 7 = 3(x^2 + 2xh + h^2) + 7 = 3x^2 + 6xh + 3h^2 + 7$

$\underline{f(x+h) = 3x^2 + 6xh + 3h^2 + 7}$

$$\lim_{h\to 0} \frac{(3x^2 + 6xh + 3h^2 + 7) - (3x^2 + 7)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{3x^2} + 6xh + 3h^2 \cancel{+7} - \cancel{3x^2} \cancel{-7}}{h} = \lim_{h\to 0} \frac{\cancel{h}(6x + 3h)}{\cancel{h}}$$

$$= \lim_{h\to 0} 6x + 3h$$

when $h=0$ for $6x + 3h$:

$6x + 3(0) = 6x$

$\boxed{f'(x) = 6x}$

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17. b) $x = 3$

$f(3) = 3(3)^2 + 7 = 3(9) + 7 = 27 + 7 = 34$

$\underline{(3, 34)}$

$x = 3$

$f'(3) = 6(3) = 18$ $\quad \underline{m = 18}$

$y - 34 = 18(x - 3)$

$y - 34 = 18x - 54$

$\quad +34 \qquad\qquad +34$

$\boxed{y = 18x - 20}$

19. a) $f(x) = 3x^2 - 2x + 3$

$f(x+h) = 3(x+h)^2 - 2(x+h) + 3 = 3(x^2 + 2xh + h^2) - 2x - 2h + 3$

$f(x+h) = 3x^2 + 6xh + 3h^2 - 2x - 2h + 3$

$$\lim_{h \to 0} \frac{(3x^2 + 6xh + 3h^2 - 2x - 2h + 3) - (3x^2 - 2x + 3)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{3x^2} + 6xh + 3h^2 \cancel{- 2x} - 2h \cancel{+3} \cancel{- 3x^2} \cancel{+ 2x} \cancel{- 3}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(6x + 3h - 2)}{\cancel{h}} = \lim_{h \to 0} 6x + 3h - 2$$

$$\boxed{f'(x) = 6x - 2}$$

when $h = 0$ for $6x + 3h - 2$:

$6x + 3(0) - 2 = 6x - 2$

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19. b) $x = 1$

$f(1) = 3(1)^2 - 2(1) + 3 = 3(1) - 2 + 3 = 3 + 1 = 4$

$(1, 4)$

$x = 1$

$f'(1) = 6(1) - 2 = 6 - 2 = 4$

$m = 4$

$y - 4 = 4(x - 1)$

$y - 4 = 4x - 4$

$ + 4 \qquad + 4$

$$\boxed{y = 4x}$$

21. a) $f(x)=\frac{-8}{x}$ $f(x+h)=\frac{-8}{(x+h)}$

$$\lim_{h\to 0}\frac{\left(\frac{-8}{x+h}\right)\frac{\cdot x}{\cdot x}-\left(\frac{-8}{x}\right)\frac{\cdot(x+h)}{\cdot(x+h)}}{h}=\lim_{h\to 0}\frac{-8x+8(x+h)}{x(x+h)}\div h$$

$$=\lim_{h\to 0}\frac{\cancel{-8x}+\cancel{8x}+8h}{x(x+h)}\cdot\frac{1}{h}=\lim_{h\to 0}\frac{8\cancel{h}}{x(x+h)}\cdot\frac{1}{\cancel{h}}=\lim_{h\to 0}\frac{8}{x(x+h)}$$

when $h=0$ for $\frac{8}{x(x+h)}$:

$$\frac{8}{x(x+0)}=\frac{8}{x^2}$$

$$\boxed{f'(x)=\frac{8}{x^2}}$$

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21. b) $x=-3$

$f(-3)=\frac{-8}{-3}=\frac{8}{3}$ $(-3, 8/3)$

$x=-3$

$f'(-3)=\frac{8}{(-3)^2}=\frac{8}{9}$ $m=8/9$

$$y-8/3=8/9(x-(-3))$$

$$y-8/3=\frac{8}{9}(x+3)$$

$$y-\frac{8}{3}=\frac{8}{9}x+\frac{8}{3}$$

$$+8/3 \qquad +8/3$$

$$\boxed{y=\frac{8}{9}x+\frac{16}{3}}$$

23. a) $f(x) = \frac{-3}{x}$ $f(x+h) = \frac{-3}{x+h}$

$$\lim_{h\to 0} \frac{\left(\frac{-3}{x+h}\right)\frac{\cdot x}{\cdot x} - \left(\frac{-3}{x}\right)\frac{\cdot(x+h)}{\cdot(x+h)}}{h} = \lim_{h\to 0} \frac{\frac{-3x+3(x+h)}{x(x+h)}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{-3x}+\cancel{3x}+3h}{x(x+h)} \div h = \lim_{h\to 0} \frac{3\cancel{h}}{x(x+h)} \cdot \frac{1}{\cancel{h}} = \lim_{h\to 0} \frac{3}{x(x+h)}$$

when $h=0$ for $\frac{3}{x(x+h)}$:

$$\frac{3}{x(x+0)} = \frac{3}{x(x)} = \frac{3}{x^2}$$

$$\boxed{f'(x) = \frac{3}{x^2}}$$

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23. b) $x=2$

$f(2) = \frac{-3}{2}$

$(2, -3/2)$

$x=2$

$f'(2) = \frac{3}{(2)^2} = \frac{3}{4}$

$m = 3/4$

$$y-\left(-\frac{3}{2}\right) = \frac{3}{4}(x-2)$$

$$y + \frac{3}{2} = \frac{3}{4}x - \frac{3}{2}$$

$-3/2$ $-3/2$

$$y = \frac{3}{4}x - \frac{6}{2}$$

$$\boxed{y = \frac{3}{4}x - 3}$$

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25. a) $\underline{s(t) = -2t^2 + 30t + 5}$

$$s(t+h) = -2(t+h)^2 + 30(t+h) + 5$$

$$s(t+h) = -2(t^2 + 2th + h^2) + 30t + 30h + 5$$

$$\underline{s(t+h) = -2t^2 - 4th - 2h^2 + 30t + 30h + 5}$$

$$\lim_{h \to 0} \frac{s(t+h) - s(t)}{h} = \lim_{h \to 0} \frac{(-2t^2 - 4th - 2h^2 + 30t + 30h + 5) - (-2t^2 + 30t + 5)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{-2t^2} - 4th - 2h^2 \cancel{+30t} + 30h \cancel{+5} \cancel{+2t^2} \cancel{-30t} \cancel{-5}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(-4t - 2h + 30)}{\cancel{h}} = \lim_{h \to 0} -4t - 2h + 30$$

$$\boxed{s'(t) = -4t + 30}$$

when $h = 0$ for $-4t - 2h + 30$:

$$-4t - 2(0) + 30 = -4t + 30$$

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25. b) $s'(t) = -4t + 30$

$$s'(2) = -4(2) + 30 = -8 + 30 = \boxed{22}$$

The function s(t) is the height, or position, function. By finding the derivative of that function we find the function for the velocity/speed. The input t is still the time, or number of seconds past while the output, s'(t), is the meters travelled per second at that time.

$\boxed{\text{After 2 seconds, the rocket is traveling at a velocity/speed of 22 meters per second.}}$

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27. a) $s(t) = 50 - 2t^2$ $s(t+h) = 50 - 2(t+h)^2$

$s(t+h) = 50 - 2(t^2 + 2th + h^2) = 50 - 2t^2 - 4th - 2h^2$

$s(t+h) = 50 - 2t^2 - 4th - 2h^2$

$$\lim_{h \to 0} \frac{s(t+h) - s(t)}{h} = \lim_{h \to 0} \frac{(50 - 2t^2 - 4th - 2h^2) - (50 - 2t^2)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{50} - \cancel{2t^2} - 4th - 2h^2 - \cancel{50} + \cancel{2t^2}}{h} = \lim_{h \to 0} \frac{\cancel{h}(-4t - 2h)}{\cancel{h}}$$

$$= \lim_{h \to 0} -4t - 2h$$

when $h = 0$ for $-4t - 2h$:

$-4t - 2(0) = -4t$

$\boxed{s'(t) = -4t}$

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27. b) $s'(t) = -4t$

$s'(1) = -4(1) = \boxed{-4}$

After 1 second, the pebble is traveling at a speed, or velocity, of -4 meters per second. The negative sign indicates the pebble is traveling downwards.

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29. a) $P(x) = 45x - 0.0025x^2 - 5000$

$P(x+h) = 45(x+h) - 0.0025(x+h)^2 - 5000$

$= 45x + 45h - 0.0025(x^2 + 2xh + h^2) - 5000$

$= 45x + 45h - 0.0025x^2 - 0.005xh - 0.0025h^2 - 5000$

$P(x+h) = 45x + 45h - .0025x^2 - .005xh - .0025h^2 - 5000$

$$\lim_{h \to 0} \frac{(45x + 45h - .0025x^2 - .005xh - .0025h^2 - 5000) - (45x - .0025x^2 - 5000)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{45x} + 45h \cancel{- .0025x^2} - .005xh - .0025h^2 \cancel{- 5000} \cancel{- 45x} \cancel{+ .0025x^2} \cancel{+ 5000}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(45 - .005x - .0025h)}{\cancel{h}} = \lim_{h \to 0} 45 - .005x - .0025h$$

when $h = 0$ for $45 - .005x - .0025h$:

$45 - .005x - .0025(0) = 45 - .005x$

$$\boxed{P'(x) = 45 - .005x}$$

29. b) $P'(x) = 45 - .005x$

$P'(800) = 45 - .005(800) = 45 - 4 = 41$

$\boxed{P'(800) = 41}$

The derivative of a profit function is the function for the marginal profit, which is a function that gives the profit of selling the next item.

$\boxed{\text{After selling 800 car seats, the profit from selling the next car seat would be \$41.}}$

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31. a) $C(x) = x^2 + 40x + 800$

$C(x+h) = (x+h)^2 + 40(x+h) + 800 = x^2 + 2xh + h^2 + 40x + 40h + 800$

$C(x+h) = x^2 + 2xh + h^2 + 40x + 40h + 800$

$$\lim_{h \to 0} \frac{(x^2 + 2xh + h^2 + 40x + 40h + 800) - (x^2 + 40x + 800)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 + \cancel{40x} + 40h + \cancel{800} - \cancel{x^2} - \cancel{40x} - \cancel{800}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(2x + h + 40)}{\cancel{h}} = \lim_{h \to 0} 2x + h + 40$$

when $h = 0$ for $2x + h + 40$:

$2x + (0) + 40 = 2x + 40$

$C'(x) = 2x + 40$

31. b) $C'(x) = 2x + 40$

$$C'(30) = 2(30) + 40 = 60 + 40 = 100$$

$$\boxed{C'(30) = 100}$$

The derivative of the cost function, is a function for the marginal cost. The marginal cost function gives the increase in cost of producing the next item.

> After producing 30 chairs, the increase in cost from selling the next chair would be $100.